Closing Thursday: 2.2
Closing Tuesday: 2.3 (part 1)

Recall

The solution(s) to $ax^2 + bx + c = 0$

are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The *x-coordinate of the vertex* of

$$y = ax^2 + bx + c$$

is: $x = -\dfrac{b}{2a}$

Entry Task: Let $f(x) = x^2 - 6x + 5$.

(a) Does it open upward or downward?
(b) What is the y-intercept?
(c) What are the x-intercepts?
(d) What are the x- and y-coordinates of the vertex?

(a) $\underbrace{a = 1}$, $b = -6$, $c = 5$

POSITIVE ⇒ $\boxed{\text{OPENS UPWARD}}$

(b) $f(0) = (0)^2 - 6(0) + 5 = \boxed{5}$

↑ y-intercept

(c) x-intercepts ⇒ $y = 0$

⇒ $x^2 - 6x + 5 = 0 \xrightarrow{\text{OR}} x = \dfrac{6 \pm \sqrt{6^2 - 4(1)(5)}}{2}$

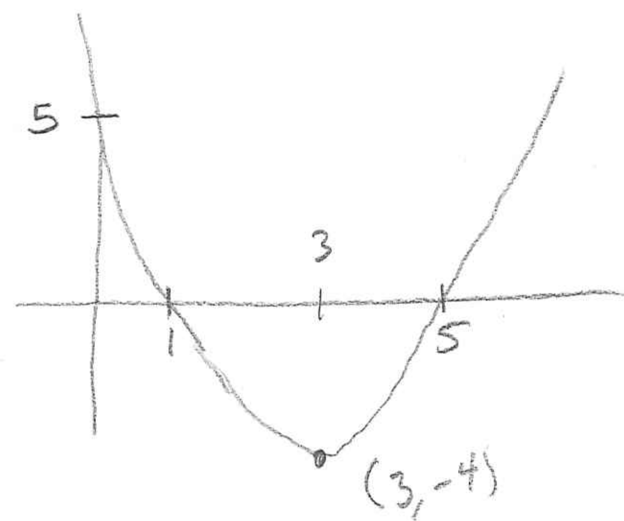
FACTOR $(x-1)(x-5) = 0$

$\boxed{x = 1}$ OR $\boxed{x = 5}$

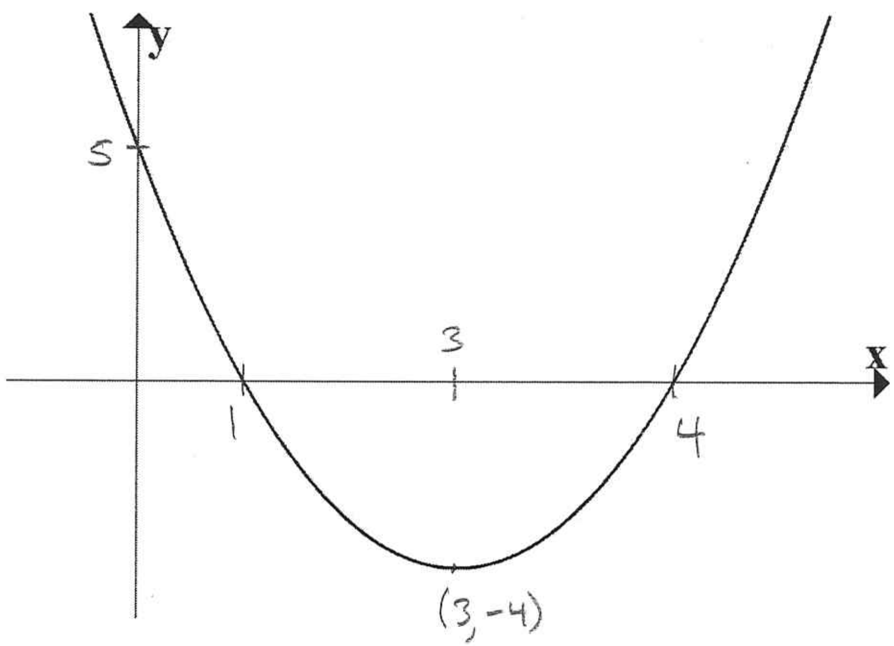
$x = \dfrac{6 \pm \sqrt{16}}{2} = \dfrac{6 \pm 4}{2}$ ⇒ 1, 5

(d) $x = -\dfrac{b}{2a} = -\dfrac{-6}{2(1)} = 3$

$y = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4$



Graph of $f(x) = x^2 - 6x + 5$.



To do applied quadratic problems

1. Write down & simplify the given quadratic function that the question is asking about.

2. Sketch a picture of the parabola. Label the vertex and the zeros.

3. Read and answer the question.

2.2: Rates and Functional Notation

Recall: Given $y = f(x)$

$$\text{average rate} = \frac{f(b)-f(a)}{b-a}$$

$$\text{overall average rate} = \frac{f(x)-f(0)}{x}$$

$$\text{diagonal slope} = \frac{f(x)}{x}$$

Now we will use functional notation!

Example: (HW 2.2 / Problem 1)
Let $f(x) = -5x - x^2$.
Find the average rate of change from x = -6 to x = 8.

WANT

$$\frac{f(8)-f(-6)}{8-(-6)}$$

$$= \frac{[-5(8)-(8)^2]-[-5(-6)-(-6)^2]}{8-(-6)}$$

$$= \frac{[-40-64]-[30-36]}{14}$$

$$= \frac{[-104]-[-6]}{14}$$

$$= -\frac{98}{14} = \boxed{-7}$$

ASIDE: $f(A) = -5A - A^2$

$f(W+4) = -5(W+4) - (W+4)^2$

$f(\text{BLAH}) = -5(\text{BLAH}) - (\text{BLAH})^2$

THIS IS HOW FUNCTIONAL NOTATION WORKS!

Example: (HW 2.2 / Prob 4-6)

Let $f(x) = 2 + x + x^2$.

(a) What is $f(x+h)$?

$$= 2 + (x+h) + (x+h)^2$$

$$= 2 + x + h + (x+h)(x+h)$$

$$= 2 + x + h + x^2 + \underbrace{xh + hx}_{2xh} + h^2$$

$$= \boxed{2 + x + h + x^2 + 2xh + h^2}$$

(b) What is $\dfrac{f(x+h)-f(x)}{h}$?

$$= \frac{[2 + (x+h) + (x+h)^2] - [2 + x + x^2]}{h}$$

DON'T FORGET!

$$= \frac{[2 + x + h + x^2 + 2xh + h^2] - [2 + x + x^2]}{h}$$

ALL SHOULD CANCEL!

$$= \frac{\cancel{2} + \cancel{x} + h + \cancel{x^2} + 2xh + h^2 - \cancel{2} - \cancel{x} - \cancel{x^2}}{h}$$

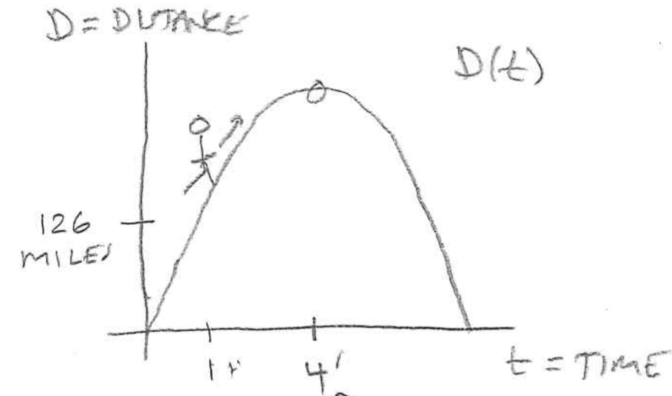
$$= \frac{h + 2xh + h^2}{h}$$

$$= \frac{h}{h} + \frac{2xh}{h} + \frac{h^2}{h}$$

$$= \boxed{1 + 2x + h}$$

Example 1: A object is launched in the air. Its distance, D(t), in miles above the ground at time t hours is given by

$$D(t) = 144t - 18t^2$$



WARM UP (*general graph questions*)

(a) How far does it go in the first hour?

(b) Give the largest interval over which the distance is increasing.

(a) $D(1) = 144(1) - 18(1)^2 = 144 - 18 = \boxed{126 \text{ MILES}}$

(b) STOPS INCREASING WHEN IT REACHES ITS VERTEX!

$$t = -\frac{b}{2a} = -\frac{144}{2(-18)} = \frac{144}{36} = 4 \text{ HOURS}$$

$\boxed{\text{INCREASING FROM } t = 0 \text{ to } t = 4}$

Again, $D(t) = 144t - 18t^2$

OVERALL RATES:

(a) Find ATS at t = 4.

(b) Find the formula for ATS(t).

(c) When will ATS be 100 mph?

AVE. SPEED FROM 0 TO 4

(a) $ATS(4) = \frac{D(4)}{4} = \frac{144(4) - 18(4)^2}{4} = \frac{288 \text{ MILES}}{4 \text{ HOURS}} = \boxed{72 \text{ mph}}$

(b) $ATS(t) = \frac{D(t)}{t} = \frac{144(t) - 18(t)^2}{t} = \frac{144t}{t} - \frac{18t^2}{t} = 144 - 18t$

$\boxed{ATS(t) = 144 - 18t}$ CHECK $ATS(4) = 144 - 18(4) = 72$ ✓

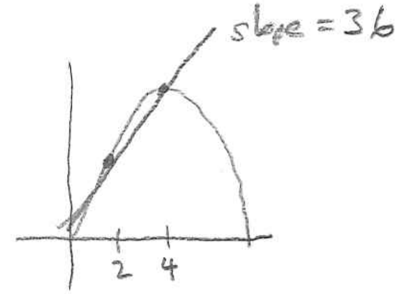
"SIMPLIFIED LINEAR FUNCTION" $(mt + b)$

(c) $ATS(t) \stackrel{?}{=} 100 \Rightarrow 144 - 18t \stackrel{?}{=} 100$

$\Rightarrow 44 = 18t$

$\Rightarrow t = \frac{44}{18} \approx \boxed{2.44 \text{ Hours}}$

Again, $D(t) = 144t - 18t^2$

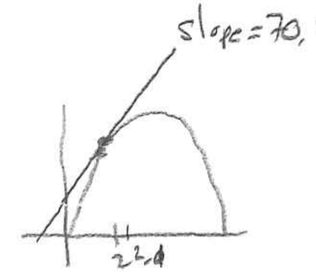


INCREMENTAL RATES:

(a) Find the average speed over the 2 hour interval starting at $t = 2$.

$$\frac{D(4) - D(2)}{4 - 2} = \frac{[144(4) - 18(4)^2] - [144(2) - 18(2)^2]}{2}$$

$$= \frac{288 - 216}{2} = \frac{72 \text{ MILES}}{2 \text{ HOURS}} = \boxed{36 \text{ mph}}$$



(b) Find the average speed over the 0.1-hour interval starting at $t = 2$.

$$\frac{D(2.1) - D(2)}{2.1 - 2} = \frac{[144(2.1) - 18(2.1)^2] - [144(2) - 18(2)^2]}{0.1}$$

$$= \frac{223.02 - 216}{0.1} = \frac{7.02 \text{ MILES}}{0.1 \text{ HOURS}} = \boxed{70.2 \text{ mph}}$$

(c) Find the general formula for the average speed over the 0.1-hour interval starting at *t*.

$$\frac{D(t+0.1)-D(t)}{t+0.1-t} = \frac{[144(t+0.1)-18(t+0.1)^2]-[144t-18t^2]}{0.1}$$

$$= \frac{\cancel{144t}+14.4-18(t^2+0.2t+0.01)-\cancel{144t}+18t^2}{0.1}$$

$$= \frac{14.4-\cancel{18t^2}-3.6t-0.18+\cancel{18t^2}}{0.1} = \frac{14.22-3.6t}{0.1}$$

$$= \boxed{142.2-36t}$$

gives this

h = 0.1

(d) Find the general formula for the average speed over the *h*-hour interval starting at *t*.

$$\frac{D(t+h)-D(t)}{h} = \frac{[144(t+h)-18(t+h)^2]-[144t-18t^2]}{h}$$

$$= \frac{\cancel{144t}+144h-18(t^2+2th+h^2)-\cancel{144t}+18t^2}{h}$$

$$= \frac{144h-\cancel{18t^2}-36th-18h^2+\cancel{18t^2}}{h} = \frac{144h}{h}-\frac{36th}{h}-\frac{18h^2}{h}$$

$$= \boxed{144-36t-18h}$$